JEE MAIN 2024

Sample Paper - 5

Time Allowed: 3 hours **Maximum Marks: 300**

General Instructions:

- All questions are compulsory.
- There are three parts and each part carries 30 questions where the first 20 questions are MCQs and the next 10 questions are numerical.
- Section-A within each part is compulsory. Attempt any 5 questions from section-B within each part.
- You will get 4 marks for each correct response and 1 mark will be deducted for an incorrect answer. However, there is no negative marking for Section-B (Numerical Questions)

PHYSICS (Section-A)

1. The main scale of a vernier callipers marked upto 10 cm is equally divided into 100 equal parts. Its vernier scale of 10 divisions coincides with 9 mm on the main scale. The least count of the instrument is **[4]**

   a) 0.001 cm
   b) 0.01 cm
   c) 0.002 cm
   d) 0.02 cm

2. A car accelerates from rest at a constant rate of 2 m/s^2 for some time. Then, it retards at a constant rate of 4 m/s^2 and comes to rest. If the total time for which it remains in motion is 3 seconds, what is the total distance travelled? **[4]**

   a) 2 m
   b) 6 m
   c) 4 m
   d) 3 m

3. At a height 0.4 m from the ground, the velocity of a projectile in vector form is: $\vec{v}$ = (6$\hat{i}$ + 2 $\hat{j}$) m/s. The angle of projection is: (g = 10m/s^2) **[4]**

   a) $\tan^{-1}(3/4)$
   b) 60°
   c) 45°
   d) 30°

4. Two small balls of the same size and masses m_1 and m_2 ($m_1 > m_2$) are tied by a thin weightless thread and dropped from a certain height. Taking upward buoyancy force F into account the tension T of the thread during the flight after the motion of the ball becomes uniform will be: **[4]**

   a) $(m_1 - m_2)g$
   b) $\left[\frac{(m_1-m_2)}{2}\right] g$
   c) $(m_1 + m_2)g$
   d) $\left[\frac{(m_1+m_2)}{2}\right] g$

5. A car of mass m starts from rest and accelerates so that the instantaneous power delivered to the car has a constant magnitude P_0. The instantaneous velocity of this car is proportional to **[4]**

a) $t^2 P_0$

b) $\frac{t}{\sqrt{m}}$

c) $t^{\frac{-1}{2}}$

d) $t^{\frac{1}{2}}$

6. A particle is moving in the X-Y plane with a constant velocity along a line parallel to the x-axis away from the origin. The magnitude of its angular momentum about the origin: **[4]**

a) remains constant

b) is zero

c) goes on decreasing

d) goes on increasing

7. For rain harvesting of water, in a residential complex, an underground water tank of a square cross-section of each side 500 m is constructed. If the water level in the month of August inside the tank was 420 m, what would be the difference between the thrust experienced by the vertical surface of the tank and the bottom of the tank? (Take g = 10 m/s^2) **[4]**

a) 10.3×10^{11} N

b) 14×10^{11} N

c) 8.1×10^{11} N

d) 6.1×10^{11} N

8. The radius of a metal sphere at room temperature T is R and the coefficient of linear expansion of the metal is a. The sphere is heated a little by a temperature ΔT so that its new temperature is (T + ΔT). The increase in the volume of the sphere is approximate: **[4]**

a) $2\pi R\alpha\Delta T$

b) $\pi R^2\alpha\Delta T$

c) $\frac{4\pi R^3\alpha\Delta T}{3}$

d) $4\pi R^3\alpha\Delta T$

9. In a diesel engine the cylinder compresses air from S.T.P. to about $\frac{1}{5}^{\text{th}}$ of the original volume and a pressure of 25 atmosphere. The temperature of compressed air is nearly **[4]**

a) 135 K

b) 1365 K

c) 580 K

d) 853 K

10. A particle executes simple harmonic motion (amplitude = A) between x = -A and x = +A. The time taken for it to go from 0 to $\frac{A}{\sqrt{2}}$ is T_1 and to go from $\frac{A}{\sqrt{2}}$ to A is T_2. Then: **[4]**

a) $T_1 = T_2$

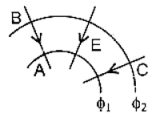
b) $T_1 > T_2$

c) $T_1 = 2T_2$

d) $T_1 < T_2$

11. In moving from A to B along an electric field line, the electric field does 6.4×10^{-19} J of work on an electron. If ϕ_1, ϕ_2 are equipotential surfaces, then the potential difference (V_C - **[4]**

V_A) is:



a) 64V

b) 4V

c) -4V

d) zero

12. A length of wire carries a steady current. It is bent first to form a circular plane coil of one turn. The same length is now bent more sharply to give a double loop of a smaller radius. The magnetic field at the centre caused by the same current is: **[4]**

i. a quarter of its first value

ii. unaltered

iii. four times of its first value

iv. half of its first value

a) iii

b) i

c) ii

d) iv

13. The hysteresis cycle for the material of transformer core is: **[4]**

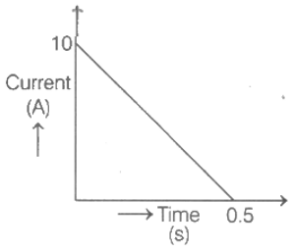
a) tall and wide

b) tall and narrow

c) short and narrow

d) short and wide

14. In a coil of resistance 100Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is **[4]**



a) 275 Wb

b) 250 Wb

c) 200 Wb

d) 225 Wb

15. A resistance of 20 Ω is connected to a source of an alternating potential V = 220 sin (100π t). The time taken by the current to change from its peak value to rms value is: **[4]**

a) 25×10^{-3} s

b) 2.5×10^{-3} s

c) 0.2 s

d) 0.25 s

16. The dimensions of $\frac{E}{B}$ are same as that of: **[4]**

a) acceleration

b) charge

c) velocity

d) current

17. A photocell is illuminated by a small bright source placed 1 m away. When the same source of light is placed ($\frac{1}{2}$) m away, the .number of electrons emitted by photocathode would: **[4]**

a) increase by a factor of 4

b) increase by a factor of 2

c) decrease by a factor of 4

d) decrease by a factor of 2

18. The total energy of an electron in an atom in an orbit is -3.4 eV. Its kinetic and potential energies are, respectively: **[4]**

a) -3.4 eV, -3.4 eV

b) 3.4 eV, 3.4 eV

c) 3.4 eV, -6.8 eV

d) -3.4 eV, -6.8 eV

19. A chain reaction is continuous due to: **[4]**

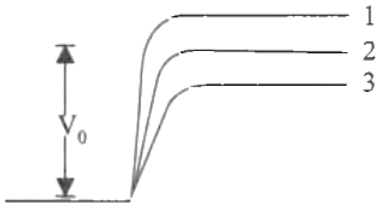
a) production of more neutrons during fission

b) large energy

c) large mass defect

d) daughter nuclei formed

20. In the figure, V_0 is the potential barrier across a p-n junction, when no battery is connected across the junction. **[4]**



a) 1 and 3 both correspond to forward bias of junction.

b) 3 and 1 both correspond to reverse bias of junction.

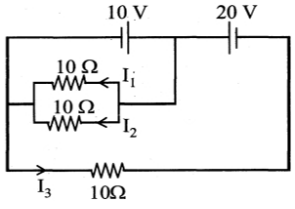
c) 1 corresponds to forward bias and three corresponds to reverse bias of junction.

d) 3 corresponds to forward bias of junction and one corresponds to reverse bias of junction.

PHYSICS (Section-B)

Attempt any 5 questions

21. In the given circuit, the value of $\left|\frac{I_1+I_3}{I_2}\right|$ is _____ **[4]**



22. Three charges $1\mu C, 1\mu C$ and $2\mu C$ are kept at vertices A, B and C of an equilateral triangle ABC of side 10 cm respectively. The resultant force on the charge at C is: **[4]**

23. A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of 3.0×10^{-2} T. The maximum emf induced the coil will be _____ $\times 10^{-2}$volt (rounded off to the nearest integer) **[4]**

24. The initial velocity v_i required to project a body vertically upward from the surface of the earth to reach a height of 10 R, where R is the radius of the earth, may be described in terms of escape velocity v such that $v_i = \sqrt{\frac{x}{y}} \times v_e$. The value of x will be _____. **[4]**

25. A rectangular block of mass 5 kg attached to a horizontal spiral spring executes simple harmonic motion of amplitude 1 m and time period 3.14 s. The maximum force exerted by spring on block is _____ N. **[4]**

26. If luminous efficiency of a lamp is 2 lumen/watt and its luminous intensity is 42 candela, then power of the lamp is _____W. **[4]**

27. Two long parallel wires carrying currents 8 A and 15 A in opposite directions are placed at a distance of 7 cm from each other. A point P is at equidistant from both the wires such that the lines joining the point P to the wires are perpendicular to each other. The magnitude of magnetic field at P is _____ $\times 10^{-6}$ T. (Given : $\sqrt{2}$ = 1.4) **[4]**

28. A one metre long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound in air at STP is 300 m/s, the frequency difference between the fundamental and second harmonic of this pipe is _____Hz. **[4]**

29. If the length of a cylinder on heating increases by 2%, the area of its base will increase by _____%. **[4]**

30. A metal wire of length 0.5 m and cross-sectional area $10^{-4}m^2$ has breaking stress $5 \times 10^8 \mathrm{Nm}^{-2}$. A block of 10kg is attached at one end of the string and is rotating in a horizontal circle. The maximum linear velocity of block will be _____ ms^{-1}. **[4]**

CHEMISTRY (Section-A)

31. If E_1, E_2 and E_3 represent respectively the kinetic energies of an electron, an alph particle and a proton respectively each having same de Broglie wavelength then: **[4]**

a) $E_1 > E_2 > E_3$ b) $E_1 > E_3 > E_2$

c) $E_1 = E_2 = E_3$ d) $E_2 > E_3 > E_1$

32. Which bond is expected to be the least polar? **[4]**

a) Si-N b) B-Cl

c) O-F d) P-F

33. The hybridisation of sulphur in sulphur dioxide is **[4]**

a) sp b) sp^2

c) dsp^2

d) sp^3

34. For the following two reactions: **[4]**

i. $CH_{4(g)} + 2O_{2(g)} \longrightarrow CO_{2(g)} + 2H_2O_{(g)}$, ΔH = -890.4 kJ

ii. $Cr_2O_{3(s)} \longrightarrow 2Cr_{(s)} + \frac{3}{2}O_{2(g)}$; ΔH = +1130 kJ

Which among the following statements is CORRECT?

a) Reaction (i) is exothermic and (ii) is endothermic

b) Both reactions are endothermic

c) Both reactions are exothermic

d) Reaction (i) is endothermic and (ii) is exothermic

35. Consider a chemical reaction $A_{(g)} + B_{(g)} \rightleftharpoons C_{(g)}$, for which K is 100. If the above equation is multiplied by 2, the value of K becomes _____. **[4]**

a) 10,000

b) 50

c) 200

d) 1000

36. Reducing sugars are sometimes characterized by a number R_{cu}, which is defined as the number of mg of copper reduced by 1 g sugar involving the half-reaction: **[4]**

$Cu^{2+} + OH^- \longrightarrow Cu_2O + H_2O$

What is the R_{cu} for 43.2 mg of carbohydrate which was oxidised by an excess of $K_3Fe(CN)_6$. The $Fe(CN)_6^{4-}$ formed in the reaction required 5.29 mL of 0.0345 N Ce^{4+} itself reduced to Ce^{3+}?

a) 269

b) 0.269

c) 2.69×10^{-2}

d) 26.9×10^{-2}

37. Which of the following statement is FALSE? **[4]**

a) Al is a light metal.

b) Al can be drawn into sheets and wire.

c) Al reacts vigorously with concentrated nitric acid.

d) Al is a good conductor of heat and electricity.

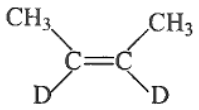
38. The most stable resonating structure of $CH_3 - \ddot{\underset{\cdot\cdot}{O}} - CH = CH$ is: **[4]**

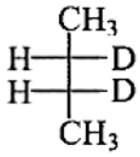
a) $CH_3 - \overset{\oplus}{O} = CH - \overset{\ominus}{C}H_2$

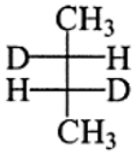
b) $CH_3 - \ddot{\underset{\cdot\cdot}{O}} - \overset{\ominus}{C}H = \overset{\oplus}{C}H_2$

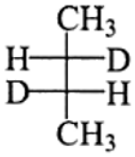
c) $CH_3 - O - \overset{\oplus}{C}H - \overset{\ominus}{C}H_2$

d) $CH_3 - \overset{\oplus}{\ddot{\underset{\cdot\cdot}{O}}} = CH - \overset{\oplus}{C}H_2$

39.  Product: **[4]**

a) 

b) 

c) 

d) All of these

40. Which of the following is the correct representation of relative lowering of vapour pressure? **[4]**

a) $\frac{P^\circ - P}{P^\circ}$

b) $\frac{P^\circ}{\Delta P} = \frac{P^\circ - P}{P^\circ}$

c) $\frac{\Delta P}{P^\circ} = \frac{P^\circ - P}{P^\circ}$

d) $\frac{P^\circ}{P^\circ - P}$

41. For an ideal solution of two components A and B, which of the following is true? **[4]**

a) $\Delta H_{mixing} < 0$ (zero)

b) A-A, B-B and A-B interactions are identical

c) $\Delta H_{mixing} > 0$ (zero)

d) A-B interaction is stronger than A-A and B-B interactions

42. Copper becomes green when exposed to moist air for a long period. This is due to: **[4]**

a) the formation of a layer of cupric hydroxide on the surface of copper.

b) the formation of basic copper sulphate layer on the surface of the metal.

c) the formation of a layer of cupric oxide on the surface of copper.

d) the formation of a layer of basic carbonate of copper on the surface of copper.

43. The decomposition of a substance **P** takes place according to first-order kinetics. Its initial concentration is reduced to one fourth in 24 s. The rate constant of the reaction is _____. **[4]**

a) $\frac{1}{24} s^{-1}$

b) $\frac{0.693}{12} s^{-1}$

c) $\frac{0.693}{16} s^{-1}$

d) $\frac{0.693}{4} s^{-1}$

44. Select the correct statement: **[4]**

A. Complex ion $[MoCl_6]^{3-}$ is paramagnetic

B. Complex ion $[Co(en)_3]^{3+}$ is diamagnetic

C. Both Complex ion $[MoCl_6]^{3-}$ is paramagnetic and Complex ion $[Co(en)_3]^{3+}$ is diamagnetic are correct

D. None of these

a) Only (D)

b) Only (A)

c) Only (C)

d) Only (B)

45. Which of the following name of compounds are matched correctly against their molecular formula? **[4]**

a) CaS_2O_3-Calcium thiosulphate

b) $Na_2P_2O_5$-Sodium pyrophosphate

c) $K_2S_2O_7$-Potassium thionate

d) NaN_3-Sodium nitride

46. Which of the following would act as flexidentate ligand? **[4]**

a) SO_4^{2-}

b) NH_2-NH_2

c) CH^-

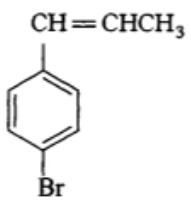
d) Ethylenediamine

47. The reaction of $C_6H_5CH{=}CHCH_3$ with HBr Produces **[4]**

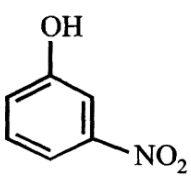
a) $C_6H_5CH_2CH_2CH_2Br$

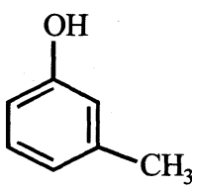
b) $C_6H_5CH_2\underset{Br}{C}HCH_3$

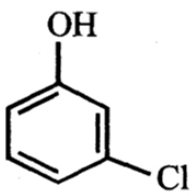
c) $C_6H_5\underset{Br}{C}HHCH_2CH_3$

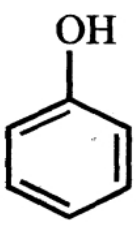
d) 

48. The strongest acid from the following is **[4]**

a) 

b) 

c) 

d) 

49. 2,4-DNP test can be used to identify **[4]**

a) Halogens

b) Aldehyde

c) Amine

d) Ether

50. In the given reaction sequence $C_6H_5 — CH_2—NH_2 \xrightarrow[\Delta]{CHCl_3/Alc.KOH} [X] \xrightarrow{H_2O/NaOH} [Y]$. [Y] will be: **[4]**

a) C_6H_5-CH_2-NH_2

b) C_6H_5NC

c) C_6H_5-CH_2OH

d) C_6H_5-CN

CHEMISTRY (Section-B)

Attempt any 5 questions

51. The number of atomic orbitals from the following having 5 radial nodes is _____. **[4]**
7s, 7p, 6s, 8p, 8d

52. **[4]**
i. $X(g) \rightleftharpoons Y(g) + Z(g)$ $K_{p1} = 3$
ii. $A(g) \rightleftharpoons 2B(g)$ $K_{p2} = 1$
If the degree of dissociation and initial concentration of both the reactants X(g) and A(g) are equal, then the ratio of the total pressure at equilibrium $\left(\frac{P_1}{P_2}\right)$ is equal to x:1. The value of x is _____. (Nearest integer)

53. Find no. of a-bonds in $NO[BF_4]$ **[4]**

54. The total number of isoelectronic species from the given set is _____. **[4]**
O^{2-}, F^-, Al, Mg^{2+}, Na^+, O^+, Mg, Al^{3+}, F

55. Complexes (ML_5) of metals Ni and Fe have ideal square pyramidal and trigonal bipyramidal geometries, respectively. The sum of the 90^o, 120^o and 180^o L- M- L angles in the two complexes is _____. **[4]**

56. The oxidation state of manganese in the product obtained in a reaction of potassium permanganate and hydrogen peroxide in basic medium is **[4]**

57. Molar mass of caffine is 194. If it contains 28.9% by mass of nitrogen, then number of atoms of nitrogen in one molecule of caffine is _____ N atom. **[4]**

58. The atomic masses of He and Ne are 4 and 20 amu, respectively. The value of the de-Broglie wavelength of He gas at -73 °C is M times that of the de-Broglie wavelength of Ne at 727°C. M is **[4]**

59. Acrylonitrile is used to manufacture polymer for synthetic fibre $H_2C = CH - CN$ (Acrylonitrile) **[4]**
Find total number of hybrid orbitals of carbon used for bonding.

60. At constant volume, 4 mol of an ideal gas when heated from 300 K to 500 K changes its internal energy by 5000 J. The molar heat capacity at constant volume is _____. **[4]**

MATHEMATICS (Section-A)

61. Let f, g : $\mathbb{N}$ - {1} $\rightarrow \mathbb{N}$ be functions defined by f(a) = α, where α is the maximum of the powers of those primes p such that p^α divides a, and g(a) = a + 1, for all a $\in \mathbb{N}$ - {1}. Then, **[4]**

the function f + g is

a) onto but not one-one

b) both one-one and onto

c) one-one but not onto

d) neither one-one nor onto

62. If the equations $2ax^2 - 3bx + 4c = 0$ and $3x^2 - 4x + 5 = 0$ have a common root, then $\frac{5a+b}{b+6c}$ is equal to (where a, b, c $\in$ R - {0}): **[4]**

a) 5

b) 1

c) -1

d) 3

63. If ${}^nC_4,\ {}^nC_5\ and\ {}^nC_6$ are in AP, then n can be **[4]**

a) 14

b) 9

c) 12

d) 11

64. The coefficient of the term independent of x in the expansion of $(1 + x + 2x^3)\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$ is **[4]**

a) $\frac{19}{54}$

b) $\frac{1}{4}$

c) $\frac{1}{3}$

d) $\frac{17}{54}$

65. If in a $\triangle$ABC, cos A + 2cos B + cos C = 2, then sin A, sin B, sin C are in: **[4]**

a) H.P

b) increasing order

c) A.P.

d) G.P

66. Let f be a differentiable function on $(0, \infty)$ and suppose that $\lim_{x\to\infty}$ (f(x) + f'(x)) = L where L is a finite quantity, then which of the following must be true? **[4]**

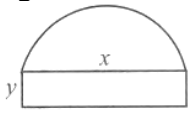
a) $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to\infty} f'(x) = L$

b) $\lim_{x\to\infty} f(x) = L$ and $\lim_{x\to\infty} f'(x) = 0$

c) $\lim_{x\to\infty} f(x) = \frac{L}{2}$ and $\lim_{x\to\infty} f'(x) = \frac{L}{2}$

d) nothing definite can be said

67. A Norman window is shown in the figure. (i.e. It is a window in which a rectangle is surmounted by a semi-circle.) If the window has a constant perimeter then for maximum light to be admitted **[4]**



a) y = 4x

b) 2x = y

c) x = 2y

d) x = 4y

68. Let $I_1 = \int_0^{\pi/2} e^{-x^2} \sin(x)\, dx$; $I_2 = \int_0^{\pi/2} e^{-x^2} dx$; $I_3 = \int_0^{\pi/2} e^{-x^2} (1 + x)\, dx$ and consider the statements **[4]**

I. $I_1 < I_2$

II. $I_2 < I_3$

III. $I_1 = I_3$

which of the following is (are) true?

a) I only

b) Both I and II

c) II only

d) Neither I nor II nor III

69. Locus of the point of intersection of the tangents at the ends of the normal chords of the parabola $y^2 = 4ax$ is : **[4]**

a) $(x + 2a)y^2 + 4a^3 = 0$

b) $(x + 3a)y^3 - 4a^2 = 0$

c) $(2a + x)y^2 + 4a^3 = 0$

d) $(x + 2a)y^2 + 4a^2 = 0$

70. If the circles $x^2 + y^2 - 16x - 20y + 164 = r^2$ and $(x - 4)^2 + (y - 7)^2 = 36$ intersect at two distinct points, then **[4]**

a) $1 < r < 11$

b) $r = 11$

c) $0 < r < 1$

d) $r > 11$

71. The shortest distance between the line $y = x$ and the curve $y^2 - x - 2$ is **[4]**

a) $\frac{7}{4\sqrt{2}}$

b) 2

c) $\frac{7}{8}$

d) $\frac{11}{4\sqrt{2}}$

72. The general solution of the differential equation, $\sin 2x\left(\frac{dy}{dx} - \sqrt{\tan x}\right) - y = 0$, is: **[4]**

a) $y\sqrt{\cot x} = x + c$

b) $y\sqrt{\tan x} = \cot x + c$

c) $y\sqrt{\cot x} = \tan x + c$

d) $y\sqrt{\tan x} = x + c$

73. If (a, b, c) is the image of the point (1, 2, -3) in the line, $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then a + b + c is equals to: **[4]**

a) -1

b) 1

c) 2

d) 3

74. If p^{th} q^{th}, r^{th} terms of a G.P. are the positive numbers a, b, c respectively then angle between the vectors $(\log a^2)\,\hat{i} + (\log b^2)\,\hat{j} + (\log c^2)\,\hat{k}$ and $(q-r)\hat{i} + (r-p)\hat{j} + (p-q)\hat{k}$ is: **[4]**

a) $\frac{\pi}{2}$

b) $\cos^{-1}\left(\frac{pqr}{\sqrt{p^2+q^2+r^2}}\right)$

c) $\frac{\pi}{3}$

d) $\sin^{-1}\left(\frac{1}{\sqrt{a^2+b^2+c^2}}\right)$

75. Let a random variable X have a binomial distribution with mean 8 and variance 4. If $P(X \leq 2) = \frac{k}{2^{16}}$, then k is equal to **[4]**

a) 121

b) 1

c) 137

d) 17

76. One counter is selected at random from 60 counters numbered 01, 02, ..., 60, then the probability that the sum of digits is 6, given that the product of these digits is odd, equals: **[4]**

a) $\frac{7}{25}$

b) $\frac{3}{25}$

c) $\frac{2}{7}$

d) $\frac{7}{100}$

77. If $\frac{\pi}{2} < \alpha < \pi$, then $\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha}} + \sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}}$ is equal to: **[4]**

a) -2 sec α

b) 2 cos α

c) -2 cos α

d) 2 sec α

78. If a directrix of a hyperbola centred at the origin and passing through the point (4, $-2\sqrt{3}$) is $5x = 4\sqrt{5}$ and its eccentricity is e, then **[4]**

a) $4e^4 - 24e^2 + 27 = 0$

b) $4e^4 - 24e^2 + 35 = 0$

c) $4e^4 - 12e^2 - 27 = 0$

d) $4e^4 + 8e^2 - 35 = 0$

79. If A and B are subsets of a set X, then what is $\{A \cap (X - B)\} \cup B$ equal to? **[4]**

a) $A \cap B$

b) A

c) B

d) $A \cup B$

80. If system of linear equations $(a-1)x + z = \alpha$, $x + (b-1)y = \beta$ where $a, b, c \in I$ does not have a unique solution, then maximum possible value $|a + b + c|$ is: **[4]**

a) 3

b) 1

c) 0

d) 4

MATHEMATICS (Section-B)

Attempt any 5 questions

81. Let AD and BC be two vertical poles at A and B respectively on horizontal ground. If AD = 8 m, BC = 11 m and AB = 10 m; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is _____. **[4]**

82. Let g(x) = $f\left[\frac{x}{f(x)}\right]$ where f(x) is a differentiable positive function on (0, ∞) such that f(1) = $f'(1)$. Determine $g'(1)$ **[4]**

83. The direction ratios of two lines L_1 and L_2 are < 4, -1,3 > and < 2, -1, 2 > respectively. A vector $\vec{V}$ is perpendicular to L_1 and L_2 both such that $|\vec{V}| = 15$. If $\vec{V} = x_1\hat{i} + x_2\hat{j} + x_3\hat{k}$ then find the value of $|x_1 + x_2 + x_3|$. **[4]**

84. Let A be the area of the region {(x, y): $y \geq x^2$, $y \geq (1 - x)^2$, $y \leq 2x(1 - x)$}. Then 540 A is equal to **[4]**

85. Let volume of tetrahedron ABCD be cubic units and the volume of parallelepiped whose three coterminous edges are line segments joining the centroid of any face of tetrahedron with centroids of the other faces is $\frac{p}{q}$, where p and q are co-prime, then find the value of (p - q). **[4]**

86. The probability that an event A happens in one trial of an experiment, is 0.4. Three independent trials of the experiments are performed. The probability that event A happens at least once, is _____. **[4]**

87. Let a, b be two non-zero real numbers. If p and r are the roots of the equation x^2 - 8ax + 2a = 0 and q and s are the roots of the equation x^2 + 12bx + 6b = 0, such that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$, are in A.P., Then $a^{-1} - b^{-1}$ is equal to _____. **[4]**

88. If in a $\triangle$ABC, it is given that sin C = $\frac{\sqrt{2}-\sin A}{\cos A}$ and one side of triangle is 2, then find the ratio of maximum possible area of triangle to minimum possible area of triangle. **[4]**

89. Let S be the set containing all 3 $\times$ 3 matrices with entries from {-1, 0, 1}. The total number of matrices A $\in$ S such that the sum of all the diagonal elements of A^T A is 6 is _____. **[4]**

90. Let d be the number of integers in the range of the function f(x) = **[4]**

$$\begin{cases} 4, & if \quad -4 \leq x < -2 \\ |x|, & if \quad -2x \leq x < 7 \\ \sqrt{x}, & if \quad 7 \leq x < 14 \end{cases}$$

. Also roots of P(x) = x^2 + mx - 4 m + 20 are α and β. If $\alpha < \frac{d-3}{4} < \frac{d-3}{2} < \beta$ and the smallest integral value of m is k, then find the value of (k - 5).

JEE MAIN 2024

Sample Paper - 5

Solution

PHYSICS (Section-A)

1.

(b) 0.01 cm

Explanation: 1 M.S.D. = $\frac{10}{100}$cm = 0.10 cm;

1 V.S.D. = $\frac{09}{10}$cm = 0.09 cm

∴ L.C. = 1 M.S.D. - 1 V.S.D.

= 0.10 - 0.09 = 0.01 cm

2.

(b) 6 m

Explanation: t_1 = 2 s and t_2 = 1 s

Now, $s = ut + (\frac{1}{2})\, at^2$

∴ $s_1 = 0 + (\frac{1}{2}) \times 2 \times 4 = 4$m

and $s_2 = (\frac{1}{2}) \times 4 \times 1 = 2$ m

∴ $s_1 + s_2 = 6$ m

3.

(d) 30°

Explanation: $v^2 = u^2 - 2gh$

or $v^2 = u^2 + 2gh$

or $u_x^2 + u_y^2 = v_x^2 + v_y^2 + 2gh$

As $v_x = u_x$

∴ $u_y^2 = v_y^2 + 2gh$

or $u_y^2 = (2)^2 + 2 \times 10 \times 0.4 = 12$

∴ $u_y = \sqrt{12} = 2\sqrt{3}\text{m/s}$

and $u_x = v_x = 6$ m/s

∴ $\tan\theta = \frac{u_y}{u_x} = \frac{2\sqrt{3}}{6} = \frac{1}{\sqrt{3}}$

∴ θ = 30°

4.

(b) $\left[\frac{(m_1 - m_2)}{2}\right] g$

Explanation:

Since, the sizes of both the balls are same the force of buoyancy will be same. In equilibrium,

$2F = m_1 g + m_2 g$

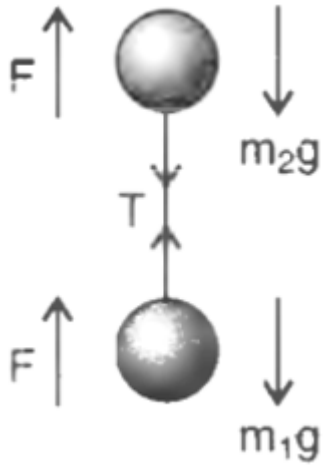
or, $F = \left(\frac{m_1 + m_2}{2}\right) g$

Now, considering the equilibrium of lower ball,

$T + F = m_1 g$

or $T = m_1 g - F$

$= \left[m_1 - \left(\frac{m_1+m_2}{2}\right)\right] g = \left(\frac{m_1-m_2}{2}\right) g$



5. **(d)** $t^{\frac{1}{2}}$

Explanation: Constant power of the car is

$P_0 = Fv = m\frac{dv}{dt}v$

$\Rightarrow mvdv = P_0dt$

$\Rightarrow \int mvdv = \int P_0dt$

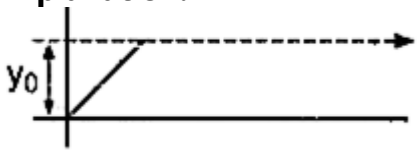
$\Rightarrow \frac{1}{2} mv^2 = P_0t$

$\therefore v = \sqrt{\frac{2P_0t}{m}}$

$\Rightarrow v \propto t^{\frac{1}{2}}$

6. **(a)** remains constant

Explanation:



Mathematically speaking* angular momentum is moment of momentum about the origin. The angle goes on decreasing from 90°. But it is the perpendicular distance to line of motion x mv, which is angular momentum. This is a constant. Therefore, the answer expected is that the angular momentum is a constant. (For rectilinear motion, can one discuss angular momentum? But let us take it only as a mathematical exercise)

7. **(d)** 6.1×10^{11} N

Explanation: As liquid pressure depends on the height of liquid, pressure due to liquid is not same throughout over the vertical surface of the tank.

$\therefore$ Average pressure on the vertical surface $P_{avg} = \frac{(0+h\rho g)}{2} = \frac{h\rho g}{2}$

$\therefore$ Thrust on the vertical surface = $P_{avg} \times$ area

$= \frac{\rho g h^2 l}{2}$

The area in the above relation is not the area of square cross-section but the area of vertical surface experiencing thrust due to water pressure.

Across the bottom, pressure due to liquid is same throughout,

$\therefore$ Pressure on the bottom $P_b = h\rho g$

$\therefore$ Thrust on the bottom surface = $P_b \times$ area

$= h\rho g l^2$

∴ Difference in thrust = $h\rho gl^2 - \frac{h^2\rho gl}{2}$

$= 420 \times 10^3 \times 10 \times 25 \times 10^4 - \frac{(420)^2 \times 10^3 \times 10 \times 500}{2}$

$= 10.5 \times 10^{11} - 4.41 \times 10^{11}$

$= 6.09 \times 10^{11}$ N

8. **(d)** $4\pi R^3 \alpha \Delta T$

Explanation: As we know that,

As, $\gamma = \frac{\Delta V}{V \times \Delta T}$ and $\gamma = 3\alpha$

$3\alpha = \frac{\Delta V}{\left(\frac{4\pi}{3}R^3\right)\Delta T}$

which gives, $\Delta V = 4\pi R^3 \alpha \Delta T$

9. **(b)** 1365 K

Explanation: At S.T.P

T_1 = 273 K and P_1 = 1 atm

$V_2 = \frac{V_1}{5}$, P_2 = 25 atm

Being the same gas compressed, n = constant

Equation of state

$\Rightarrow \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$\therefore T_2 = \frac{P_2 V_2}{P_1 V_1} \times T_1 = \frac{25 \times V_1}{5 \times 1 \times V_1} \times 273$

$\therefore T_2$ = 1365 K

10. **(a)** $T_1 = T_2$

Explanation: Using, x = A sin ωt

For $x = \frac{A}{\sqrt{2}}$, we have, $\frac{A}{\sqrt{2}}$ = A sin ωT_1

$\therefore \sin \omega T_1 = \frac{1}{\sqrt{2}}$

$\therefore T_1 = \frac{\pi}{4\omega}$

For x = A, $\sin \omega(T_1 + T_2) = 1$

$\therefore T_1 + T_2 = \frac{\pi}{4\omega}$

$\therefore T_2 = \frac{\pi}{2\omega} - T_1 = \frac{\pi}{2\omega} - \frac{\pi}{4\omega} = \frac{\pi}{4\omega}$ i.e., $T_1 = T_2$

11. **(b)** 4V

Explanation: Electric field = 6.4×10^{-19} J work on electron q_1 and ϕ_2 potential difference

$V_C - V_A = \frac{w}{q} = \frac{6.4 \times 10^{-19}}{1.6 \times 10^{-19}}$ = 4V

$\because q = 1.6 \times 10^{-19} C$ due to electron movement.

12. **(a)** iii

Explanation: Let the radii be r_1 and r_2 respectively.

Since there are two turns of radius r_2, $r_1 = 2r_2$

Magnetic field B at the center of the coil of radius r_1 $B_1 = \frac{\mu_0 i}{2r_1} = \frac{\mu_o i}{4r_2}$

Magnetic field B at the center of the coil of radius r_2 $B_2 = 2 \times \frac{\mu_o i}{2r_2}$

$\therefore \frac{B_2}{B_1} = \frac{2 \times \frac{\mu_o i}{2r_2}}{\frac{\mu_o i}{4r_2}}$ = 4

13.

(b) tall and narrow

Explanation: The transformer core is soft iron material which has small coercivity and large retentivity. Therefore its hysteresis loop is tall and narrow.

14.

(b) 250 Wb

Explanation: Induced constant, $I = \frac{e}{R}$

Here, e = induced emf = $\frac{d\phi}{dt}$

$I = \frac{e}{R} = \left(\frac{d\phi}{dt}\right) \cdot \frac{1}{R}$

$d\phi = IRdt$

$\phi = \int IRdt$

∴ Here, R is constant

$\therefore \phi = R\int Idt$

$\int Idt$ = Area under I-t graph = $\frac{1}{2} \times 10 \times 0.5 = 2.5$

$\therefore \phi = R \times 2.5 = 100 \times 2.5 = 250\text{Wb}$

15.

(b) 2.5×10^{-3} s

Explanation: Peak value to rms value means, current becomes $\frac{1}{\sqrt{2}}$ times

If peak is at t = 0, current is of the form,

$I = I_0 \cos 100\ \pi t$

$\Rightarrow \frac{1}{\sqrt{2}} \times I_0 = I_0 \cos 100\ \pi t$

$\Rightarrow t = \frac{1}{400}$ s

$= 2.5 \times 10^{-3}$ s

16.

(c) velocity

Explanation: The dimensions of $\frac{E}{B}$ are same as that of velocity.

17. **(a)** increase by a factor of 4

Explanation: As $I \propto \frac{1}{r^2}$

$\therefore I' \propto \frac{1}{(1/2)^2}$ or $I' \propto 4$

i.e., intensity will become four times, therefore number of photons will become 4 times.

18.

(c) 3.4 eV, -6.8 eV

Explanation: In Bohr's model of H atom

$\therefore$ K.E. = $|\text{TE}| = \frac{|\text{U}|}{2}$

∴ K.E. = 3.4 eV

U = -6.8 eV

19. **(a)** production of more neutrons during fission

Explanation: Due to the production of neutrons, a chain of nuclear fission is established which continues until the whole of the source substance is consumed.

20.

(d) 3 corresponds to forward bias of junction and one corresponds to reverse bias of junction.

Explanation: Height of potential barrier decreases when the p-n junction is forward biased and it increases when the junction is reverse biased.

PHYSICS (Section-B)

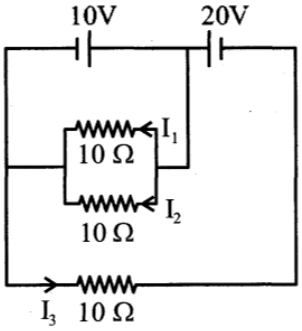
21. 2.0

Explanation:

We have $10I_1 = 10I_2 = 10V$

$\Rightarrow I_1 = I_2 = 1A$

and, $-10V + 20V - 10I_3 = 0$ [By KVL in largest loop]



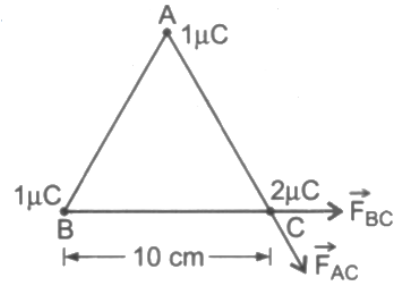
$\Rightarrow 10V = 10I_3 \Rightarrow I_3 =$ A

So, $\left|\frac{I_1+I_3}{I_2}\right| = \frac{1+1}{1} = 2$

22. 3.12

Explanation:

The resultant force on charge C is due to charge A and B, i.e.,



$\vec{F}_{AC} + \vec{F}_{BC}$

$\left|\vec{F}_{AC}\right| = \left|\vec{F}_{BC}\right| = F$

$= \frac{1}{4\pi\varepsilon_0}\frac{1\times2\times10^{-12}}{(10\times10^{-2})^2}$

$F_{\text{res}} = \sqrt{F^2 + F^2 + 2F^2\cos 60^\circ}$

$= \sqrt{3F^2} = \sqrt{3}F$

$= \sqrt{3}\frac{1}{4\pi\varepsilon_0}\frac{2\times10^{-12}}{10^{-2}}$

$= \sqrt{3}\times 9\times 10^9\times 2\times 10^{-10}$

= 3.12 N

23. 60.0

Explanation:

Given,

Magnetic field, $B = 3 \times 10^{-2}$T

Angular speed of coil, ω = 50 rad s^{-1}

Number of turns in coil, n = 20

Maximum emf, $\varepsilon = N\omega AB$

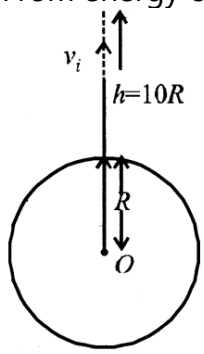
$\Rightarrow \varepsilon = 20 \times 50 \times \pi \times (0.08)^2 \times 3 \times 10^2 = 60.28 \times 10^2$

Rounded off to nearest integer = 60

24. 10.0

Explanation:

From energy conservation,



$-\frac{GM_em}{R}+\frac{1}{2}mv_i^2=-\frac{GM_em}{11R}$

$v_i=\sqrt{\frac{20}{11}\frac{GM_e}{R}}\because v_e=\sqrt{\frac{2GM_e}{R}}$

$\therefore v_i=\sqrt{\frac{10}{11}}V_e$

$\therefore$ x = 10.

25. 20

Explanation:

Given,

Mass of block, m = 5 kg

Amplitude of SHM, A = 1 m

Time period, T = 3.14 = π

$\Rightarrow$ T = $\frac{2\pi}{\omega}\Rightarrow\omega$ = 2

Maximum force,

F_{max} = m a_{max} = m (Aω^2) = mA $(2)^2$ = 5 $\times$ 1 $\times$ 4 = 20 N

26. 264

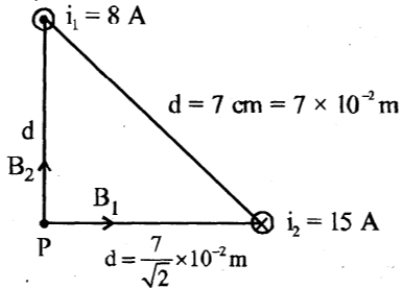
Explanation:

Luminous flux = 4πI = 4 $\times$3.14 $\times$ 42 = 528 lumen

Power of lamp = $\frac{\text{luminous flux}}{\text{luminous efficiency}}=\frac{528}{2}$ = 264 W.

27. 68.0

Explanation:



Magnetic fields due to both wires will be perpendicular to each other.

$B_1=\frac{\mu_0 i_1}{2\pi d}$ $B_2=\frac{\mu_0 i_2}{2\pi d}$

$B_{net}=\sqrt{B_1^2+B_2^2}=\frac{\mu_0}{2\pi d}\sqrt{i_1^2+i_2^2}$

$=\frac{4\pi\times10^{-7}}{2\pi\times(7/\sqrt{2})\times10^{-2}}\times\sqrt{8^2+15^2}$ = 68 $\times$ 10^{-6} T

28. 106.0

Explanation:

Given: V_{air} = 300 m/s, ρ_{gas} = 2 ρ air

Using, $V=\sqrt{\frac{B}{\rho}};\frac{V_{gas}}{V_{air}}=\frac{\sqrt{\frac{B}{2\rho_{air}}}}{\sqrt{\frac{B}{\rho_{air}}}}$

$\Rightarrow V_{gas} = \frac{V_{air}}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 150\sqrt{2}$ m/s

And f_{nth} harmonic $= \frac{nv}{2L}$ (in open organ pipe)

(L = 1 metre given)

$\therefore$ f_{2nd} harmonic - $f_{fundamental} = \frac{2v}{2\times1} - \frac{v}{2\times1} = \frac{v}{2}$

$\therefore$ f_{2nd} harmonic - $f_{fundamental} = \frac{150\sqrt{2}}{2} = \frac{150}{\sqrt{2}} \approx 106$ Hz

29. 4

Explanation:

If length increases by 2% on heating, radius will also increase by 2%.

As base area $\propto$ (radius)^2, so it will increase by 4%.

30. 50.0

Explanation:

Given, length of metal wire, ℓ = 0.5 m

Cross-sectional area, A = 10^{-4} m^2

Breaking stress = 5×10^8 Nm^{-2}

Mass of block m = 10 kg

$T_{max} = \text{Breaking stress} \times \text{Area}$

$\frac{mv^2}{\ell} = 5 \times 10^8 \times 10^{-4} = 5 \times 10^4$

$\frac{10v^2}{0.5} = 5 \times 10^4 \Rightarrow v = \sqrt{\frac{0.5\times5\times10^4}{10}} = 50$ m/s

CHEMISTRY (Section-A)

31.

(b) $E_1 > E_3 > E_2$

Explanation: $KE = \left(\frac{1}{2}\right) mu^2$ and $\lambda = \frac{h}{mu}$

$\therefore KE = \frac{1}{2}m\frac{h^2}{m^2\lambda^2} = \frac{h^2}{2m\lambda^2}$

32.

(c) O-F

Explanation: O-F

33.

(b) sp^2

Explanation: In SO_2, the Lewis-dot structure is

$O = \ddot{S} = O$

Electron pairs at S = 2 (σ-bonds) + 1 (lone-pair) = 3

sp^2 hybridised.

34. **(a)** Reaction (i) is exothermic and (ii) is endothermic

Explanation: A negative value of ΔH indicates that the reaction is exothermic, while a positive value of ΔH indicates that the reaction is endothermic.

35. **(a)** 10,000

Explanation: K = 100

$K_1 = (K)^2 = (100)^2$ = 10,000

36. **(a)** 269

Explanation: 269

37.

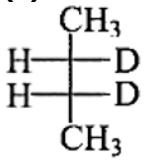
(c) Al reacts vigorously with concentrated nitric acid.

Explanation: Al reacts vigorously with concentrated nitric acid.

38. **(a)** $CH_3 - \overset{\oplus}{O} = CH - \overset{\ominus}{C}H_2$

Explanation: All atoms have complete octet structure.

39. **(a)**



Explanation: Reaction involve syn-addition of H_2.

40.

(c) $\frac{\Delta P}{P^\circ} = \frac{P^\circ - P}{P^\circ}$

Explanation: Relative lowering of vapour pressure is written as: $\frac{\Delta P}{P^\circ} = \frac{P^\circ - P}{P^\circ}$

P° = Vapour pressure of pure component

P = Vapour pressure of a component in the solution

41.

(b) A-A, B-B and A-B interactions are identical

Explanation: Solutions in which solute-solute and solvent-solvent interactions are almost similar to solute-solvent interactions are known as the ideal solutions.

42.

(d) the formation of a layer of basic carbonate of copper on the surface of copper.

Explanation: the formation of a layer of basic carbonate of copper on the surface of copper.

43.

(b) $\frac{0.693}{12} s^{-1}$

Explanation: Order of reaction = 1

Amount of the substance left after n half-lives is given as $[A]_t = \frac{[A]_0}{2^n}$

$\therefore \frac{1}{4} = \frac{1}{2^n}$

$\therefore$ n = 2

$2 \times t_{\frac{1}{2}}$ = 24s (given)

$\therefore t_{\frac{1}{2}}$ = 12s

k = $\frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{12} s^{-1}$

44.

(c) Only (C)

Explanation: $d^3, t_{2g}^{1,1,1} eg^{0,0}$ and $d^6, t_{2g}^{2,2,2} eg^{0,0}$

45. **(a)** CaS_2O_3-Calcium thiosulphate

Explanation: CaS_2O_3-Calcium thiosulphate

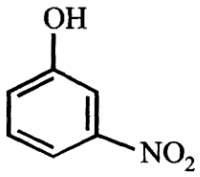
46. **(a)** SO_4^{2-}

Explanation: SO_4^{2-}

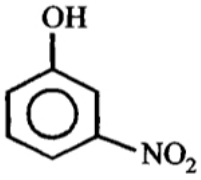
47.

(c) $C_6H_5\underset{\underset{Br}{|}}{C}HHCH_2CH_3$

Explanation: $C_6H_5\underset{\underset{Br}{|}}{C}HHCH_2CH_3$

48. **(a)** 

Explanation: Strongest acid from the following is



-NO_2 group has more EWG nature which makes the compound more acidic.

49.

(b) Aldehyde

Explanation: 2, 4-dinitro phenyl hydrazine reacts with aldehyde and ketones to form 2, 4-dinitro phenyl hydrazone derivatives. It is a condensation reaction with elimination of water.

50. **(a)** C_6H_5-CH_2-NH_2

Explanation: Ph-CH_2-NH_2 $\xrightarrow{CHCl_3+\overset{\ominus}{O}H}$ $Ph-CH-\overset{\oplus}{N}=\overset{\ominus}{C}$ $\xrightarrow{\overset{\ominus}{O}H/H_2O}$ Ph-CH_2-NH_2 + HCOOH

CHEMISTRY (Section-B)

51. 3

Explanation:

Radial mode = n - ℓ - 1

Orbital	n - ℓ - 1
7s	7 - 0 - 1 = 6
7p	7 - 1 - 1 = 5
6s	6 - 0 - 1 = 5
8p	8 - 1 - 1 = 6
8d	8 - 2 - 1 = 5

So, Answer is 3.

52. 12

Explanation:

$$x \rightleftharpoons y(g) + z(g);\ K_{p_1} = 3$$

At t = 0, a 0 0

At equilibrium, a − αa αa αa

$\therefore K_{P_1} = \frac{p_z p_y}{p_x}$; Total pressure = P_1

$= \frac{\left(\frac{\alpha}{1+\alpha}\times P_1\right)^2}{\frac{1-\alpha}{1+\alpha}P_1} = 3$, or, $\frac{\alpha^2 P_1}{(1+\alpha)(1-\alpha)} = 3$; $\therefore \frac{\alpha^2 P_1}{1-\alpha^2} = 3$

$$A \rightleftharpoons 2B \quad K_{p_2} = 1$$

At t = 0, a

At equilibrium, a − αa 2αa

$\therefore K_{P_2} = \frac{\left(\frac{2\alpha}{1+\alpha}\times P_2\right)^2}{\frac{1-\alpha}{1+\alpha}\times P_2}$ or $1 = \frac{4\alpha^2 P_2}{1-\alpha^2}$

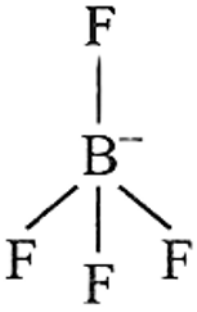
$\frac{k_{p_1}}{k_{p_2}} = \frac{P_1}{4P_2}$ or $\frac{P_1}{4P_2} = \frac{3}{1}$

$\therefore P_1 : P_2 = 12 : 1$

53. 5

Explanation:

$N \equiv \overset{+}{O}$



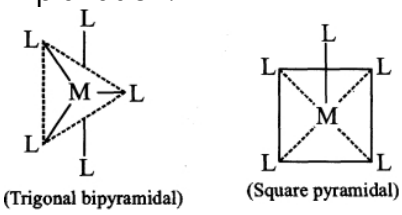
54. 5

Explanation:

Isoelectronic species O^{2-}, F^{-}, Mg^{2+}, Na^{+}, Al^{3+} (all have $10e^{-}$)

55. 20.0

Explanation:



$\angle 120^{o}$ = 3; $\angle 90^{o}$ = 6; $\angle 180^{o}$ = 2

$\angle 180^{o}$ = 1 $\Rightarrow$ Total = 10 $\Rightarrow$ Total = 10

Total number of 180^{o}, 90^{o} and 120^{o} L- M- L bond angles = 10 + 10 = 20

56. 4.0

Explanation:

$2\overset{+7}{K}\overset{+7}{Mn}O_4 + 3H_2O_2 \xrightarrow{\text{basic medium}} 2\overset{+4}{Mn}O_2 + 3O_2 + 2H_2O + 2KOH$

57. 4

Explanation:

Molar mass of caffeine = 194u

N present in one molecule of caffeine = 28.9 % of 194 = $\frac{28.9}{100} \times 194$ = 56u

Mass of one N atom = 14u

Hence 14u = 1N atom

56u = $\frac{56}{14}$N atom = 4N atom

58. 5

Explanation:

$\therefore m^2v^2 = 2mKE \therefore mv = \sqrt{2m\text{KE}}$

λ(wavelength) $= \frac{h}{mv} = \frac{h}{\sqrt{2m\text{KE}}} \propto \frac{h}{\sqrt{2m(T)}}$

Where T = Temperature in Kelvin

λ(He at -73^{o}C = 200K) $= \frac{h}{\sqrt{2\times4\times200}}$

λ(Ne at 727^{o}C = 1000 K) $= \frac{h}{\sqrt{2\times20\times1000}}$

$\therefore \frac{\lambda(\text{He})}{\lambda(\text{Ne})} = M = \sqrt{\frac{2\times20\times1000}{2\times4\times200}} = 5$

Thus M = 5

59. 8

Explanation:

$\underset{(sp^2)}{H_2C} = \underset{(sp^2)}{CH} - \underset{(sp)}{C} \equiv N$

60. 6.25

Explanation:

$\Delta U = nC_V\Delta T$

$5000 = 4 \times C_V(500 - 300)$

$C_V = 6.25\ JK^{-1}\ mol^{-1}$

MATHEMATICS (Section-A)

61.

(d) neither one-one nor onto

Explanation: f : N - {1} $\rightarrow$ N f(a) = α

Where α is max of powers of prime P such that p^{α} divides a. Also g(a) = a + 1

$\therefore$ f(2) = 1 g(2) = 3

f(3) = 1 g(3) = 4

f(4) = 2 g(4) = 5

f(5) = 1 g(5) = 6

$\Rightarrow$ f(2) + g(2) = 1 + 3 = 4

f(3) + g(3) = 1 + 4 = 5

f(4) + g(4) = 2 + 5 = 7

f(5) + g(5) = 1 + 6 = 7

$\therefore$ Many one f(x) + g(x) does not contain 1

$\Rightarrow$ into function

62.

(b) 1

Explanation: 1

63. **(a)** 14

Explanation: If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in AP, then

$2 \cdot {}^nC_5 = {}^nC_4 + {}^nC_6$

[If a, b, c are in AP , then 2b = a + c]

$\Rightarrow 2\frac{n!}{5!(n-5)!} = \frac{n!}{4!(n-4)!} + \frac{n!}{6!(n-6)!}$

$\left[\because {}^nC_r = \frac{n!}{r!(n-r)!}\right]$

$\Rightarrow \frac{2}{5\cdot4!(n-5)(n-6)!}$

$= \frac{1}{4!(n-4)(n-5)(n-6)!} + \frac{1}{6\cdot5\cdot4!(n-6)!}$

$\Rightarrow \quad \frac{2}{5(n-5)} = \frac{1}{(n-4)(n-5)} + \frac{1}{30}$

$\Rightarrow 12\ (n - 4) = 30 + n^2 - 9n + 20$

$\Rightarrow n^2 - 21n + 98 = 0$

$\Rightarrow n^2 - 14n - 7n + 98 = 0$

$\Rightarrow n(n - 14) - 7(n - 14) = 0$

$\Rightarrow (n - 7)\ (n - 14) = 0$

$\Rightarrow n = 7$ or 14

64.

(d) $\frac{17}{54}$

Explanation: t_{r+1} of $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9 = {}^9C_r \left(\frac{3}{2}x^2\right)^r \left(-\frac{1}{3x}\right)^{9-r}$

$= {}^9C_r \left(\frac{3}{2}\right)^r \left(-\frac{1}{3}\right)^{9-r} x^{3r-9}$

t_{r+1} is independent of x, if $3r - 9 = 0 \Rightarrow r = 3$

For r = 3, ${}^9C_r \left(\frac{3}{2}\right)^r \left(-\frac{1}{3}\right)^{9-r} = {}^9C_3 \left(\frac{3}{2}\right)^3 \left(-\frac{1}{3}\right)^6$

$= \frac{7}{18}$

t_{r+1} contains $\frac{1}{x^3}$, if $3r - 9 = -3 \Rightarrow r = 2$

For r = 2, ${}^9C_r \left(\frac{3}{2}\right)^r \left(-\frac{1}{3}\right)^{9-r}$

$= {}^9C_2 \left(\frac{3}{2}\right)^2 \left(-\frac{1}{3}\right)^7 = -\frac{1}{27}$

$\Rightarrow$ Coefficient of the term independent of x in the given expression = $\frac{-2}{27} + \frac{7}{18} = \frac{-4+21}{54} = \frac{17}{54}$

65.

(c) A.P.

Explanation: A.P.

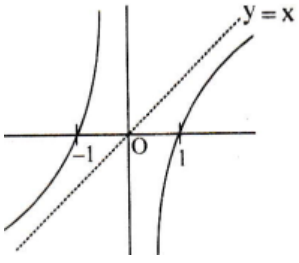
66.

(b) $\lim_{x\to\infty} f(x) = L$ and $\lim_{x\to\infty} f'(x) = 0$

Explanation:

$\therefore$ f(x) is differentiable in $(0, \infty)$

Hence, $\lim_{x\to\infty} f(x)$ must exist and is finite.



$\therefore$ y = f(x) must have a horizontal asymptote as $x \to \infty$ then only $\lim_{x\to\infty} f(x)$ will exist.

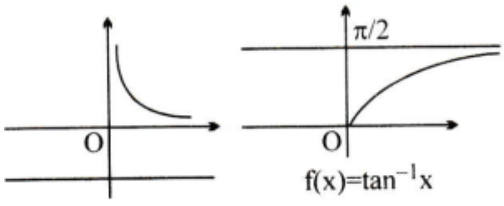
If f(x) has an inclined asymptotes as $y = x - \frac{1}{x}$ then $\lim_{x\to\infty} f(x) \to \infty$

$\therefore$ f(x) has a horizontal asymptote

Hence, $\lim_{x\to\infty} f'(x) \to 0$

$\Rightarrow$ (C) (also see figure for f(x) = $\tan^{-1}x$)

e.g., Take the example given



i. Let f(x) $= x \sin\frac{1}{x}$ which is differentiable in $(0, \infty)$

$f'(x) = \sin\frac{1}{x} - \frac{1}{x}\cos\frac{1}{x}$

$$f(x) + f'(x) = \underbrace{\left(x \sin\frac{1}{x}\right)}_{\lim_{x\to\infty} \to 1} + \frac{\left(\sin\frac{1}{x} - \frac{1}{x}\cos\frac{1}{x}\right)}{\lim_{x\to\infty} \to 0}$$

Hence, $\lim_{x\to\infty} f(x) = L$ and $\lim_{x\to\infty} f'(x) = 0$

ii. f(x) = $\tan^{-1}x$ in $(0, \infty)$

67.

(c) x = 2y

Explanation: Perimeter = $\frac{\pi x}{2}$ + 2y + x = k, constant

$\Rightarrow$ c = $(\pi + 2)x + 4y$ ...(c = 2k)

Area of the window, A

= $\frac{\pi x^2}{8}$ + xy

= $\frac{\pi x^2}{8} + x\frac{(c-(\pi+2)x)}{4}$, x > 0

= $\frac{cx}{4} - \frac{\pi x^2}{4} + \frac{(\pi-4)x^2}{8}$

= $\frac{cx}{4} - \frac{\pi x^2}{8} - \frac{x^2}{2} = \frac{cx}{4} - \frac{x^2}{8}(\pi + 4)$

$\Rightarrow \frac{dA}{dx} = \frac{c}{4} - \frac{2x}{8}(\pi + 4)$

$\frac{dA}{dx} = 0 \Leftrightarrow x = \frac{c}{\pi+4}$

$\Rightarrow$ y = $\frac{c-(\pi+2)x}{4}$

$\Rightarrow$ y = $\frac{c-\frac{(\pi+2)c}{\pi+4}}{4}$

= $\frac{2c}{4(\pi+4)}$

= $\frac{c}{2(\pi+4)}$

$\Rightarrow$ y = $\frac{x}{2}$

68.

(b) Both I and II

Explanation: Since 0 < sin x < 1 and 1 + x > 1 in (0, π/2)

Hence, $I_3 > I_2 > I_1$

$\Rightarrow$ A and B are correct $\Rightarrow$ (D)

69.

(c) $(2a + x)y^2 + 4a^3 = 0$

Explanation: $(2a + x)y^2 + 4a^3 = 0$

70. **(a)** 1 < r < 11

Explanation: Circle I is $x^2 + y^2 - 16x - 20y + 164 = r^2$

$\Rightarrow (x - 8)^2 + (y - 10)^2 = r^2$

$\Rightarrow$ C1 (8, 10) is the centre of lst circle and r_1 = r is its radius

Circle II is $(x - 4)^2 + (y - 7)^2 = 36$

$\Rightarrow$ C_2(4, 7) is the centre of 2nd circle 'and r_2 = 6 is its radius.

Two circles intersects if $|r_1 - r_2| < C_1C_2 < r_1 + r_2$

$\Rightarrow |r - 6| < \sqrt{(8-4)^2 + (10-7)^2} < r + 6$

$\Rightarrow |r - 6| < \sqrt{16 + 9} < r + 6$

$\Rightarrow |r - 6| < 5 < r + 6$

Now as, 5 < r+6 always, we have to solve only

$|r - 6| < 5 \Rightarrow -5 < r - 6 < 5$

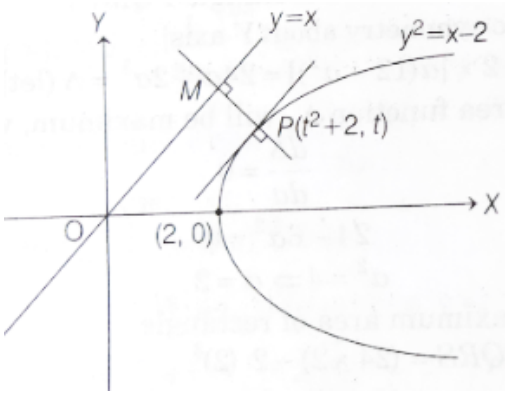
$\Rightarrow$ 6 - 5 < r < 5 + 6 $\Rightarrow$ 1 < r < 11

71. **(a)** $\frac{7}{4\sqrt{2}}$

Explanation: Given equation of curve is

y^2 = x - 2 ..... (i)

and the equation of line is



y = x ..... (ii)

Consider a point P(t^2 + 2, t) on parabola (i).

For the shortest distance between curve (i) and line (ii), the line PM should be perpendicular to line (ii) and parabola (i), i.e. tangent at P should be parallel to y = x.

$\therefore \frac{dy}{dx}\Big|_{\text{at point } P}$ = Slope of tangent at point P to curve (i) [$\because$ tangent is parallel to line y = x]

$\Rightarrow \quad \frac{1}{2y}\Big|_P = 1$ [differentiating the curve (i), we get $2y\frac{dy}{dx} = 1$]

$\Rightarrow \frac{1}{2t} = 1 \Rightarrow t = \frac{1}{2}$ $\left[\because P(x, y) = P\left(t^2 + 2, t\right)\right]$

So, the point P is $\left(\frac{9}{4}, \frac{1}{2}\right)$

Now, minimum distance = PM = $\frac{\left|\frac{9}{4} - \frac{1}{2}\right|}{\sqrt{2}}$

[$\because$ distance of a point P(x_1, y_1) from a line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$]

$= \frac{7}{4\sqrt{2}}$ units

72. **(a)** $y\sqrt{\cot x}$ = x + c

Explanation: Given, sin 2x$\left(\frac{dy}{dx} - \sqrt{\tan x}\right) - y$ = 0

or, $\frac{dy}{dx} = \frac{y}{\sin 2x} + \sqrt{\tan x}$

or, $\frac{dy}{dx} - y$ cosec^2x = $\sqrt{\tan x}$ ...(i)

Now, integrating factor (I.F) = $e^{\int -\operatorname{cosec} 2x}$

or, I.F = $e^{-\frac{1}{2}\log|\tan x|} = e^{\log(\sqrt{\tan x})^{-1}}$

= $\frac{1}{\sqrt{\tan x}} = \sqrt{\cot x}$

Now, general solution of eq. (i) is written as

y(I.F.) = $\int$Q(I.F.) dx + c

$\therefore y\sqrt{\cot x}$ = $\int \sqrt{\tan x} \cdot \sqrt{\cot x}$ dx + c

$\therefore y\sqrt{\cot x}$ = $\int$1.dx + c

$\therefore y\sqrt{\cot x}$ = x + c

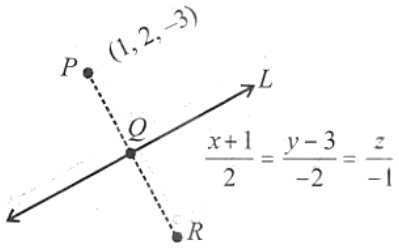
73.

(c) 2

Explanation:

$\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = \lambda$

Any point on line = Q(2λ -1, -2λ + 3, -λ)



∴ D.r. of PQ = [2λ - 2, - 2λ + 1, -λ + 3]
D.r. of given line = [2, -2, -1]
∵ PQ is perpendicular to line L
∴ 2(2λ - 2) - 2(-2λ + 1) - 1(-λ + 3) = 0
⇒ 4λ -4 + 4λ - 2 + λ - 3 = 0
⇒ 9λ - 9 = 0 ⇒ λ = 1
∵ Q is mid point of PR = Q = (1, 1, -1)
∴ Coordinate of image R = (1, 0, 1) = (a, b, c)
∴ a + b + c = 2

74. **(a)** $\frac{\pi}{2}$

Explanation: a = xy^{p-1}, b = xy^{q-1}, c = xy^{r-1}
where x : first term, y : common ratio
Let the given vectors be $\vec{X}, \vec{Y}$.
Let the angle between the vectors be θ. Then
(q - r) log a + (r - p) log b + (p - q) log c = $|\vec{X}||\vec{Y}|$ cos θ
$\Leftrightarrow |\vec{X}||\vec{Y}| \cos\theta$ = (q - r) log (xy^{p-1}) + (r - p) log (xy^{q-1}) + (p - q) log (xy^{r-1})
= log x[q - r + r - p + p - q] + (log y)[(p - 1)(q - r) + (q - 1)(r - p) + (r - 1)(p - q)]
= (0)logx + (0)log y = 0
$\Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$

75.

(c) 137
Explanation: Let for the given random variable 'X the binomial probability distribution have n-number of independent trials and probability of success and failure are p and q respectively. According to the question, Mean = np = 8 and variance = npq = 4
$\therefore q = \frac{1}{2} \Rightarrow p = 1 - q = \frac{1}{2}$
Now, $n \times \frac{1}{2} = 8 \Rightarrow n = 16$
$P(X = r) = {}^{16}C_r \left(\frac{1}{2}\right)^{16}$
∴ P(X ≤ 2) = P(X = 0) + P(X = 1) + P(X = 2)
$= {}^{16}C_0 \left(\frac{1}{2}\right)^{16} + {}^{16}C_1 \left(\frac{1}{2}\right)^{16} + {}^{16}C_2 \left(\frac{1}{2}\right)^{16}$
$= \frac{1+16+120}{2^{16}} = \frac{137}{2^{16}} = \frac{k}{2^{16}}$ (given)
⇒ k = 137

76.

(b) $\frac{3}{25}$
Explanation: S = {00, 01, 02, ..., 60}
Let A be the event that the sum of digits on the selected counter is 6, then
A = {06, 60, 15, 51, 24, 42, 33}
Let B be the event that the product of digits is odd, then
B = {11, 13, 15, 17, 19, 31, 33, ..., 59}

$\Rightarrow A \cap B$ = {15, 33, 51} Required probability = P(A|B) = $\frac{P(A \cap B)}{P(B)}$

= $\frac{\frac{3}{60}}{\frac{25}{60}} = \frac{3}{25}$

77. **(a)** -2 sec α

Explanation: $\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha}} + \sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}}$

= $\frac{1-\sin\alpha+1+\sin\alpha}{\sqrt{1-\sin^2\alpha}}$

= $\frac{2}{|\cos\alpha|}$ = -2 sec α ...[$\because \cos\alpha < 0$ in $\left(\frac{\pi}{2}, \pi\right)$]

78.

(b) $4e^4 - 24e^2 + 35 = 0$

Explanation: Let the equation of hyperbola is

$\frac{x^2}{a^2} - \frac{y^2}{b^2}$ = 1 ....(i)

Since, equation of given directrix is $5x = 4\sqrt{5}$

So, $5\left(\frac{a}{e}\right) = 4\sqrt{5}$ [$\because$ equation of directrix is x = $\frac{a}{e}$]

$\Rightarrow \frac{a}{e} = \frac{4}{\sqrt{5}}$ ..(ii)

and hyperbola (i) passes through point $(4,-2\sqrt{3})$

So, $\frac{16}{a^2} - \frac{12}{b^2} = 1$ ...(iii)

The eccentricity e = $\sqrt{1 + \frac{b^2}{a^2}}$

$\Rightarrow e^2 = 1 + \frac{b^2}{a^2}$

$\Rightarrow a^2e^2 - a^2 = b^2$ ....(iv)

From Equation (ii) and (iv), we get

$\frac{16}{5}e^4 - \frac{16}{5}e^2 = b^2$ ...(v)

From Eqs. (ii) and (iii), we get

$\frac{16}{\frac{16}{5}e^2} - \frac{12}{b^2} = 1 \Rightarrow \frac{5}{e^2} - \frac{12}{b^2} = 1$

$\Rightarrow \frac{12}{b^2} = \frac{5}{e^2} - 1 \Rightarrow \frac{12}{b^2} = \frac{5-e^2}{e^2}$

$\Rightarrow b^2 = \frac{12e^2}{5-e^2}$ ..(vi)

From equations (v) and (vi) we get

$16e^4 - 16e^2 = 5\left(\frac{12e^2}{5-e^2}\right) \Rightarrow 16\left(e^2-1\right)\left(5-e^2\right) = 60$

$\Rightarrow 4\left(5e^2 - e^4 - 5 + e^2\right) = 15$

$\Rightarrow 4e^4 - 24e^2 + 35 = 0.$

79.

(d) A $\cup$ B

Explanation: A $\cup$ B

80.

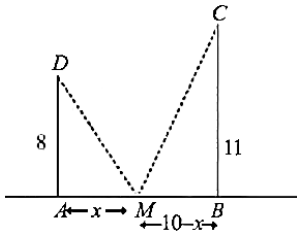
(d) 4

Explanation: 4

MATHEMATICS (Section-B)

81. 5

Explanation:



Let AM = x m

$\therefore (MD)^2 + (MC)^2 = 64 + x^2 + 121 + (10 - x)^2 = f(x)$ (say)

$f'(x) = 2x - 2(10 - x) = 0$

$\Rightarrow 4x = 20 \Rightarrow x = 5$

$f''(x) = 2 - 2(-1) > 0$

$\therefore f(x)$ is minimum at x = 5 m

82. 0

Explanation:

We have $g(x) = f\left(\frac{x}{f(x)}\right)$

On differentiating w.r.t.x, we get

$g'(x) = f'\left(\frac{x}{f(x)}\right) \times \left(\frac{f(x) - xf'(x)}{f^2(x)}\right)$

$\therefore f'(1) = f'\left(\frac{1}{f(1)}\right) \times \left(\frac{f(1) - f'(1)}{f^2(1)}\right)$

As $f(1) = f'(1)$

$\Rightarrow g'(1) = 0$

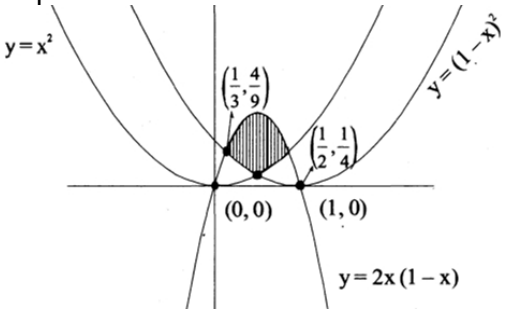
83. 15

Explanation:

15

84. 25.0

Explanation:



$A = 2\int_{\frac{1}{3}}^{\frac{1}{2}} \left(2x - 2x^2 - (1 - x)^2\right) dx$

$A = 2\left[x^2 - \frac{2x^3}{3} - \frac{(x-1)^3}{3}\right]_{\frac{1}{3}}^{\frac{1}{2}}$

$A = 2\left[\frac{1}{4} - \frac{2}{3} \times \frac{1}{8} - \frac{1}{24} - \frac{1}{9} - \frac{2}{3} \times \frac{1}{27} - \frac{8}{3 \times 27}\right] \Rightarrow \frac{5}{108}$

$540A = \frac{5}{108} \times 540 = 25$

85. 8

Explanation:

8

86. 0.784

Explanation:

Given, that, P(A) = 0.4, $P(\bar{A})$ = 0.6

P (the event A happens at least once)

= 1 - P (none of the event happens)

= 1 - (0.6 (0.6) (0.6)

= 1 - 0.216

= 0.784

87. 38.0

Explanation:

Given quadratic equations is x^2 - 8ax + 2a = 0

P + r = 8a

products of roots

pr = 2a

$\frac{1}{p} + \frac{1}{r}$ = 4

$\frac{2}{q}$ = 4

q = $\frac{1}{2}$

p = $\frac{1}{5}$

Another quadratic equation is x^2 + 12bx + 6b = 0 Sum of roots,

q + s = -12b

qs = 6b

$\frac{1}{q} + \frac{1}{s}$ = - 2

$\frac{2}{r}$ = -2

r = -1

s = $\frac{-1}{4}$

Now, $\frac{1}{a} - \frac{1}{b}, = \frac{2}{pr} - \frac{6}{qs}$ = 38

88. 2

Explanation:

2

89. 5376.0

Explanation:

Tr(AA^T) = 6

$AA^T = \begin{bmatrix} p & q & r \\ s & t & u \\ v & w & x \end{bmatrix} \begin{bmatrix} p & s & v \\ q & t & w \\ r & u & x \end{bmatrix}$

Now given $p^2 + q^2 + r^2 + s^2 + t^2 + u^2 + v^2 + w^2 + x^2$

= $^9C_3 \times 2^6$ = 5376

90. 8

Explanation:

Range of f(x) is [0, 7)

Hence, d = 7.

Now, one root of P(x) is less than 1 and other root greater than 2.

Hence, P(1) < 0 $\Rightarrow$ 21 - 3m < 0 $\Rightarrow$ m > 7

and P(2) < 0 $\Rightarrow$ 24 - 2m < 0 $\Rightarrow$ m > 12

Hence, m > 12.

$\therefore$ Least integral value of m is 13.

$\Rightarrow$ (k - 5) = 8